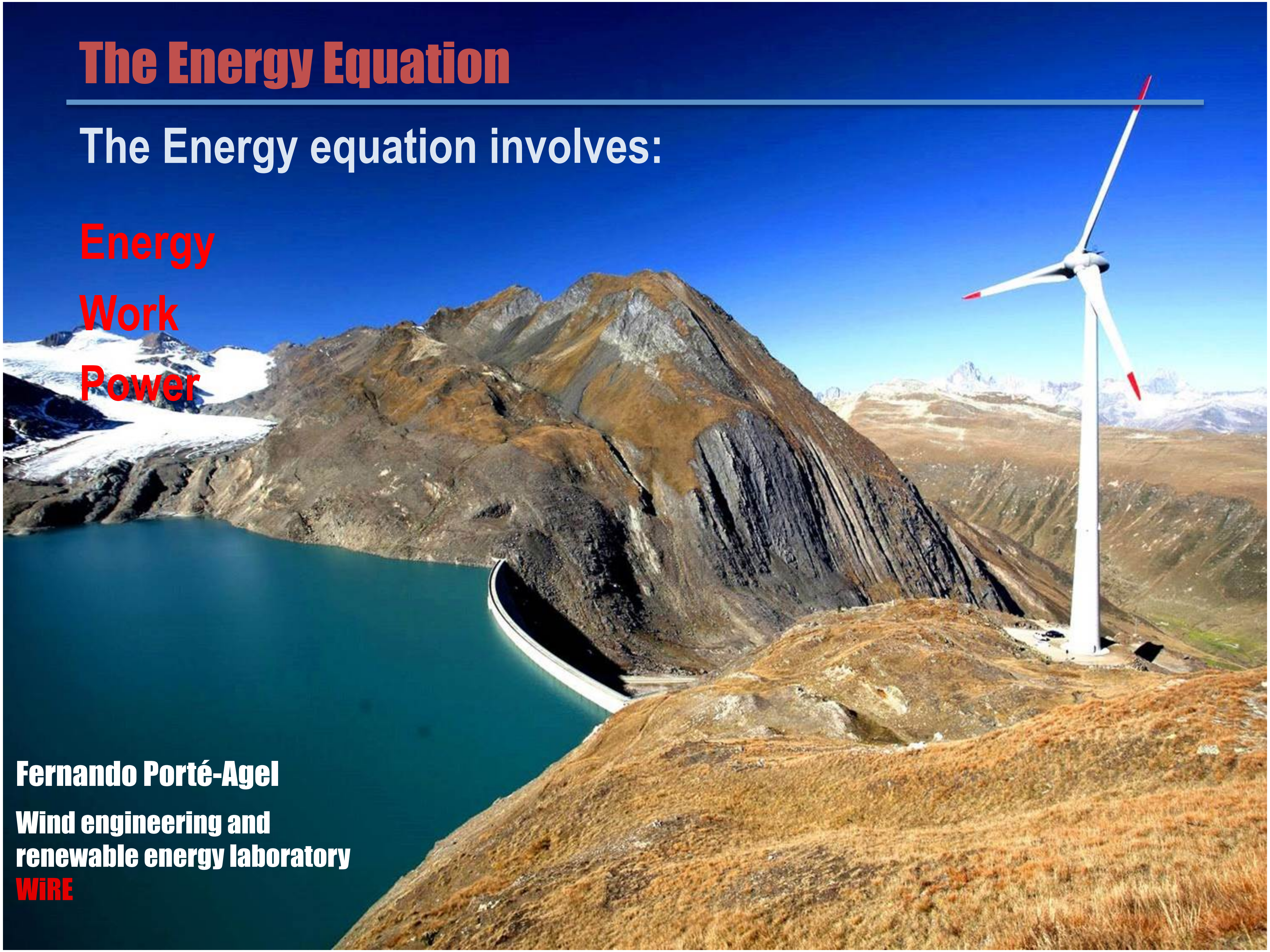
The Energy Equation

The Energy equation involves:

Energy

Work

Power



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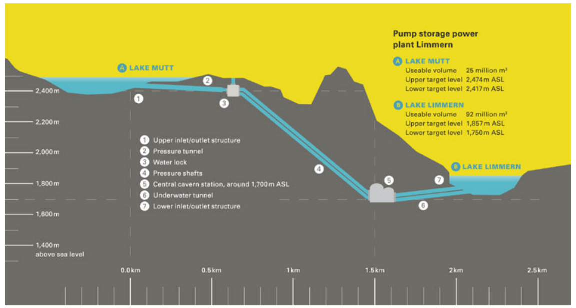
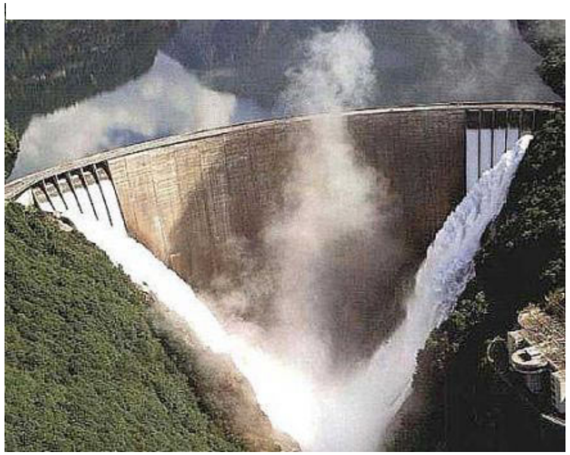
WiRE

Energy - Motivation

Examples



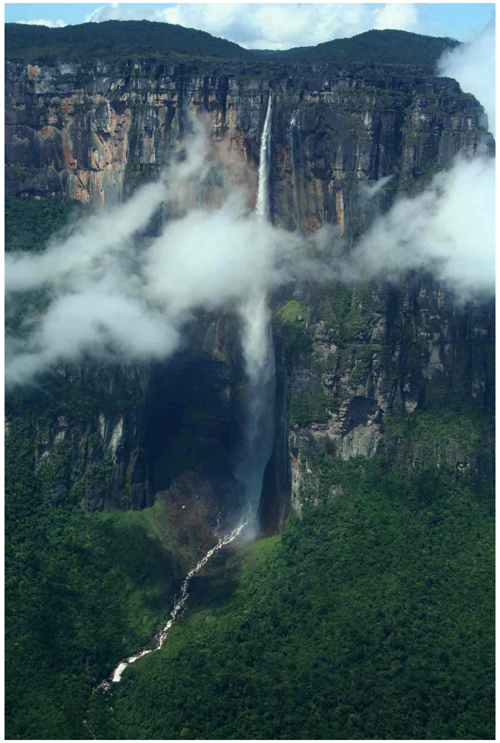


Energy

When matter has *energy*, the matter can be used to do *work.*

Some forms of energy:

Kinetic Energy | Gravitational Energy | Thermal Energy

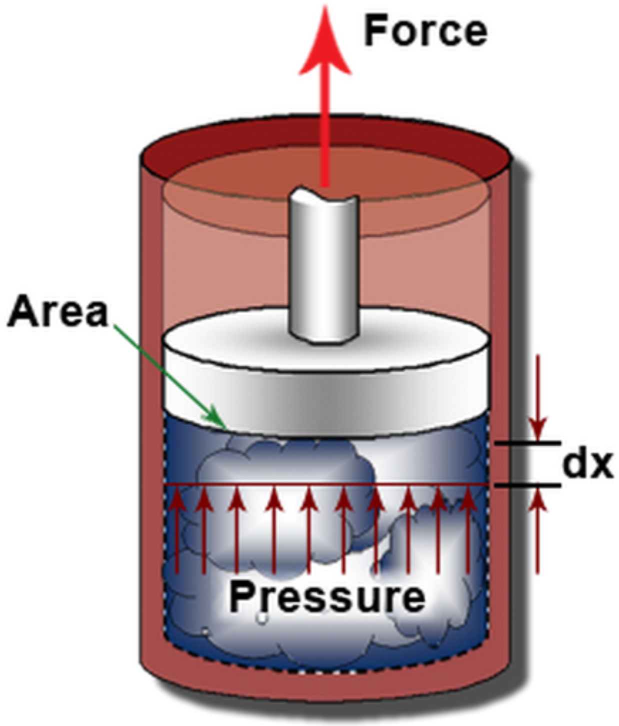


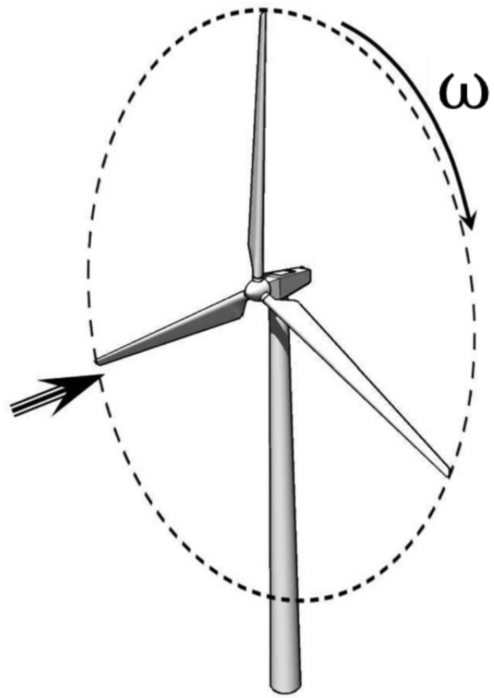
Work

Definition:

Work is **force** acting through a **distance** when the force is parallel to the direction of motion. *Units: Joules (symbol: J) J = N m*

Examples:





$$\Delta W_{flow} = F \cdot \Delta x$$

$$\Delta W_{shaft} = T \cdot \Delta \theta$$

Power

Definition:

•***Power*** expresses a rate of work or energy:

$$P \equiv \frac{quantity\ of\ work\ (or\ energy)}{interval\ of\ time} = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \dot{W}$$

•Let's assume work (ΔW) is given by the product of force and displacement:

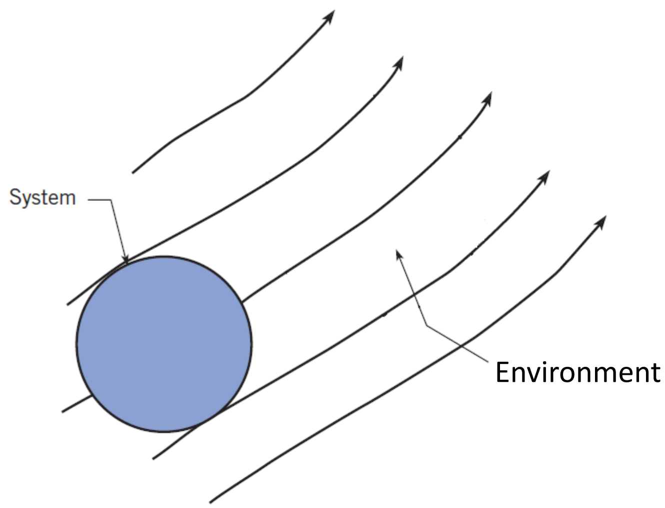
$$\Delta W = F\Delta x \longrightarrow P = \lim_{\Delta t \to 0} \frac{F\Delta x}{\Delta t} = \mathrm{FV}$$

•When a shaft is rotating, the amount of work (ΔW) is given by the product of torque and angular displacement:

$$\Delta W = T\Delta\theta \longrightarrow P = \lim_{\Delta t \to 0} \frac{T\Delta\theta}{\Delta t} = \mathrm{T}\omega$$

Units of Power: Watt (symbol: W); W = J/s

Energy Equation: General Form



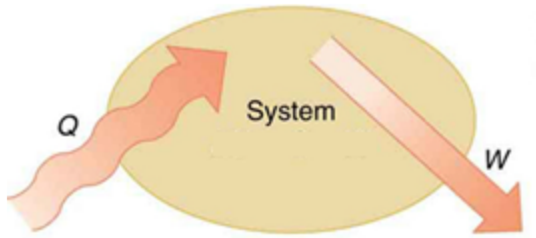
System and Environment:

•A ***system*** is a body of matter that is under consideration and it always contains the same matter.

•An imaginary boundary separates the system from all other matter, which is called the ***environment.***

Energy Equation: General Form

The Energy equation for a system is:



$$\left\{\begin{array}{c}\text{net rate of}\\ \text{thermal energy}\\ \text{entering system}\end{array}\right\} - \left\{\begin{array}{c}\text{net rate at which}\\ \text{system does work}\\ \text{on environment}\end{array}\right\} = \left\{\begin{array}{c}\text{rate of change of}\\ \text{energy of the matter}\\ \text{within the system}\end{array}\right\}$$

$$\dot{Q} - \dot{W} = \frac{dE}{dt}$$

Energy Equation: General Form

Recall Reynolds Transport Theorem:

$$\left.\frac{dB_{sys}}{dt}\right\}_{\text{Lagrangian}} = \frac{\mathrm{d}}{\mathrm{dt}}\int_{cv} b\rho d\mathbb{V} + \int_{cs} b\rho\vec{V}.\overrightarrow{dA}$$

Assuming a control volume, Let the extensive property be energy $(B_{sys} = E)$ and let $(b = e)$ to obtain:

$$\dot{Q} - \dot{W} = \frac{\mathrm{d}}{\mathrm{dt}}\int_{cv} e\rho d\mathbb{V} + \int_{cs} e\rho\vec{V}.\overrightarrow{dA}$$

e is energy per unit mass in the fluid:

$$e = e_k + e_P + u$$

e_k is the kinetic energy per unit mass

e_p is the gravitational potential energy per unit mass

u is the thermal energy (or internal energy) per unit mass

Energy Equation: General Form

Substituting $\boldsymbol{e} = \mathbf{e_k} + \mathbf{e_p} + \mathbf{u}$ in energy equation:

$$\dot{Q} - \dot{W} = \frac{\mathrm{d}}{\mathrm{dt}} \int_{cv} (e_k + e_P + u)\rho d\mathbb{V} + \int_{cs} (e_k + e_P + u)\rho \vec{V} . \overrightarrow{dA}$$

$$\dot{Q} - \dot{W} = \frac{\mathrm{d}}{\mathrm{dt}} \int_{cv} \left(\frac{V^2}{2} + gz + u\right)\rho d\mathbb{V} + \int_{cs} \left(\frac{V^2}{2} + gz + u\right)\rho \vec{V} . \overrightarrow{dA}$$

Kinetic Energy:

$$e_k = \frac{kinetic\ energy\ of\ a\ fluid\ particle}{mass\ of\ this\ fluid\ particle} = \frac{m\frac{V^2}{2}}{m} = \frac{V^2}{2}$$

Gravitational Potential Energy:

$$e_p = \frac{gravitational\ potential\ energy\ of\ a\ fluid\ particle}{mass\ of\ this\ fluid\ particle} = \frac{mgz}{m} = gz$$

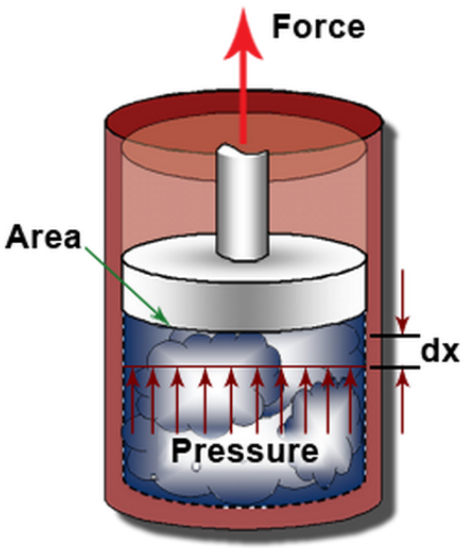
Shaft and Flow Work

Work is classified into two categories:

Flow Work

Definition:

When the force is associated with a ***pressure distribution***, then the work is called ***flow work***.



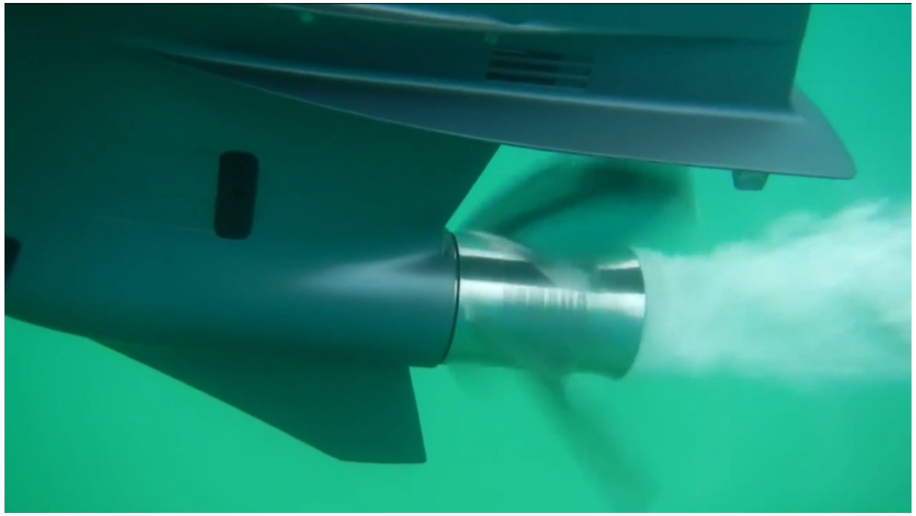
Shaft Work

Definition:

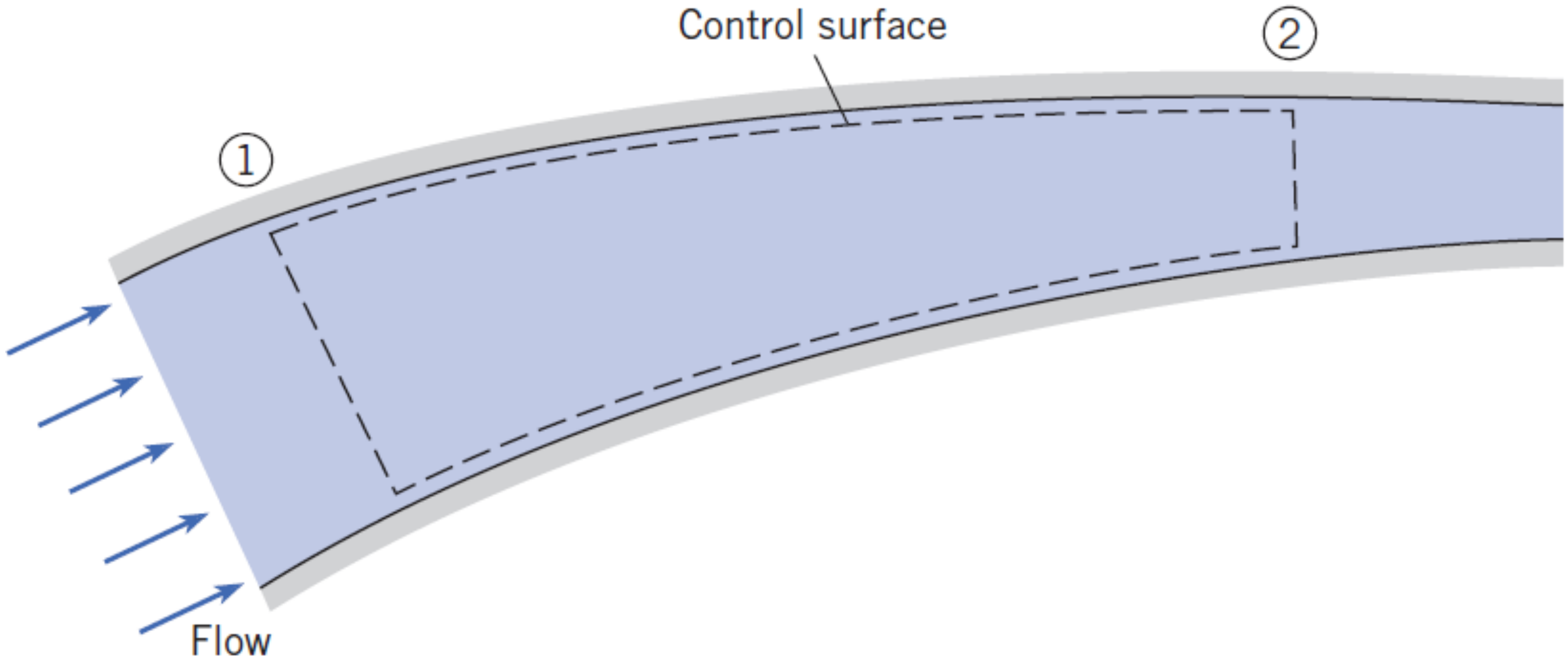
Any work that is ***not*** associated with a ***pressure force*** is called ***shaft work***.

It is usually associated with a pump or turbine.

$$\dot{W}_{shaft} = \dot{W}_{turbines} - \dot{W}_{pumps} = \dot{W}_t - \dot{W}_p$$



Flow Work



Flow Work

$$\Delta W_2 = F_2\,\Delta x_2 = (p_2 A_2)(V_2 \Delta t) \xrightarrow{\text{Rate of work}} \dot{W}_2 = \lim_{\Delta t \to 0} \frac{\Delta W_2}{\Delta t} = p_2 A_2 V_2 = \left(\frac{p_2}{\rho}\right)\dot{m}$$

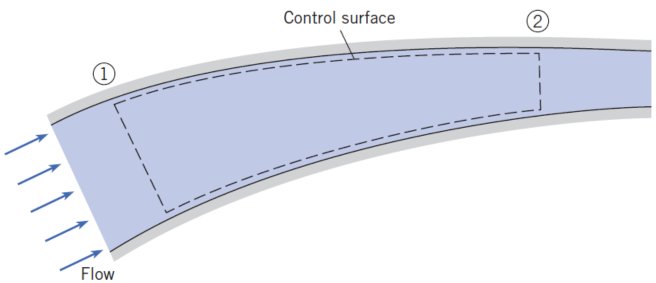
$$\dot{W}_{flow} = \dot{W}_2 + \dot{W}_1 = \left(\frac{p_2}{\rho}\right)\dot{m} - \left(\frac{p_1}{\rho}\right)\dot{m}$$

For **multiple streams of fluid** passing across a control surface:

$$\dot{W}_{flow} = \sum_{outlets} \dot{m}_{out}\left(\frac{p_{out}}{\rho}\right) - \sum_{inlets} \dot{m}_{in}\left(\frac{p_{in}}{\rho}\right)$$

The general equation for ***flow work*** is:

$$\dot{W}_{flow} = \int_{cs} \left(\frac{p}{\rho}\right)\rho\vec{V}.\overrightarrow{dA}$$



The ***work term*** is the sum of ***flow work*** and ***shaft work***:

$$\dot{W} = \dot{W}_{flow} + \dot{W}_{shaft} = \left(\int_{cs} \left(\frac{p}{\rho}\right)\rho\vec{V}.\overrightarrow{dA}\right) + \dot{W}_{shaft}$$

Energy Equation: General Form

Back to energy equation:

$$\dot{Q} - \dot{W} = \frac{\mathrm{d}}{\mathrm{dt}} \int_{cv} \left(\frac{V^2}{2} + gz + u \right) \rho d\mathbb{V} + \int_{cs} \left(\frac{V^2}{2} + gz + u \right) \rho \vec{V} . \overrightarrow{dA}$$

$$\dot{W} = \dot{W}_{flow} + \dot{W}_{shaft} = \left(\int_{cs} \left(\frac{p}{\rho} \right) \rho \vec{V} . \overrightarrow{dA} \right) + \dot{W}_{shaft}$$

$$\dot{Q} - \dot{W}_{shaft} = \frac{\mathrm{d}}{\mathrm{dt}} \int_{cv} \left(\frac{V^2}{2} + gz + u \right) \rho d\mathbb{V} + \int_{cs} \left(\frac{V^2}{2} + gz + u + \frac{p}{\rho} \right) \rho \vec{V} . \overrightarrow{dA}$$

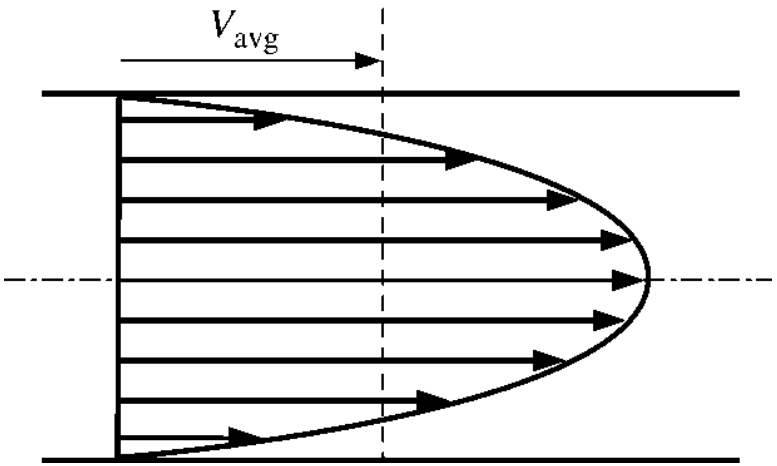
Energy Equation: General Form

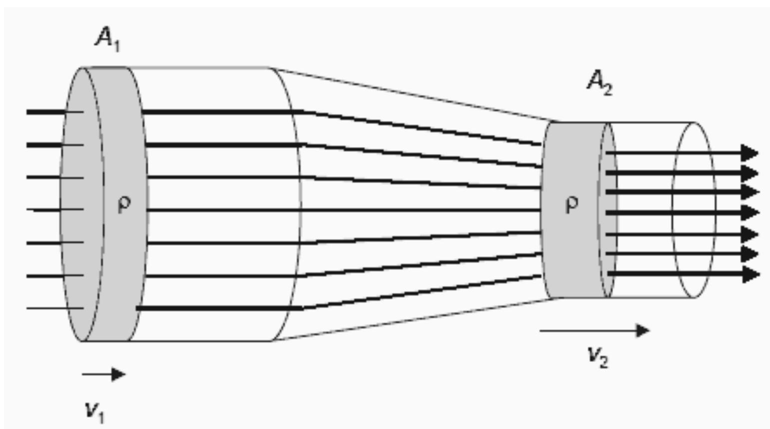
- For a series of inlet and outlet ports to a control surface with uniformly distributed velocity (V) through each port:

$$\dot{Q} - \dot{W}_s = \frac{\mathrm{d}}{\mathrm{dt}} \int_{cv} \left(\frac{V^2}{2} + gz + u \right) \rho d\mathbb{V}$$
$$+ \sum_{cs} \dot{m}_o \left(\frac{V_o^2}{2} + gz_o + u_o + \frac{p_o}{\rho} \right) - \sum_{cs} \dot{m}_i \left(\frac{V_i^2}{2} + gz_i + u_i + \frac{p_i}{\rho_i} \right)$$

Pipe Flow

Average velocity, mass and volume flow rate





Average Velocity

Definition:

$$\bar{V} = \frac{1}{A}\int_A \vec{V}.\vec{dA}$$

Volume Flow Rate

Definition:

$$Q = \int_A \vec{V}.\vec{dA}$$

Mass Flow Rate

Definition:

$$\dot{m} = \int_A \rho\vec{V}.\vec{dA}$$

For an ***incompressible*** flow:

$$\dot{m} = \rho A\bar{V} = \rho Q$$

Pipe Flow

Kinetic Energy Correction Factor

$$\left\{\begin{matrix}\textbf{\textit{Rate of KE}}\\ \textbf{\textit{transported}}\\ \textbf{\textit{across a section}}\end{matrix}\right\} = \int_A \rho\vec{V}\left(\frac{V^2}{2}\right)\overrightarrow{dA} = \int_A \frac{\rho V^3}{2}\,dA$$

α, kinetic energy correction factor definition:

$$\alpha = \frac{\text{actual KE over time that crosses a section}}{\text{KE over time by assuming a uniform velocity distribution}} = \frac{\int_A \frac{\rho V^3}{2}\,dA}{\frac{\rho\bar{V}^3 A}{2}}$$

For an ***incompressible*** flow:

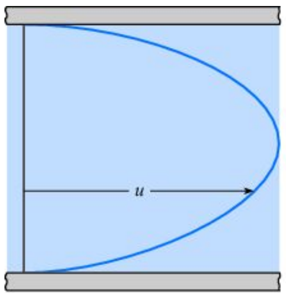
$$\alpha = \frac{1}{A}\int_A \left(\frac{V}{\bar{V}}\right)^3 dA$$

Uniform Flow $\rightarrow \alpha = 1$

Laminar Flow $\rightarrow \alpha = 2$

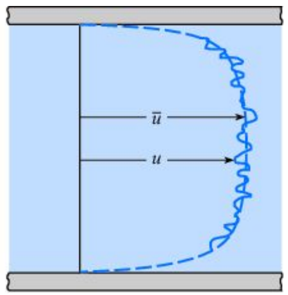
Turbulent Flow $\rightarrow \alpha \cong 1.05$

laminar



(a)

turbulent



(b)

A simplified form of the Energy Equation

$$\dot{Q} - \dot{W}_s = \frac{\mathrm{d}}{\mathrm{dt}} \int_{cv} \left(\frac{V^2}{2} + gz + u \right) \rho d\mathbb{V} + \int_{cs} \left(\frac{V^2}{2} + gz + u + \frac{p}{\rho} \right) \rho \vec{V} . \overrightarrow{dA}$$

Assuming steady Flow:

$$\dot{Q} - \dot{W}_s + \int_{A_1} \left(\frac{p_1}{\rho} + gz_1 + u_1 \right) \rho V_1 dA_1 + \int_{A_1} \frac{\rho V_1^3}{2} dA_1$$

$$= \int_{A_2} \left(\frac{p_2}{\rho} + gz_2 + u_2 \right) \rho V_2 dA_2 + \int_{A_2} \frac{\rho V_2^3}{2} dA_2$$

We can simplify $\int_{A_i} \frac{\rho V_i^3}{2} dA_i = \alpha_i \frac{\rho \bar{V}_i{}^3}{2} A_i = \alpha_i \frac{\bar{V}_i{}^2}{2} \dot{m}; \qquad i = 1,2$

Let's assume that: $\left(\frac{p_i}{\rho} + gz_i + u_i \right)$ **is constant across** A_i **for** $i = 1,2$

$$\dot{Q} - \dot{W}_s + \left(\frac{p_1}{\rho} + gz_1 + u_1 + \alpha_1 \frac{\bar{V}_1^2}{2} \right) \dot{m} = \left(\frac{p_2}{\rho} + gz_2 + u_2 + \alpha_2 \frac{\bar{V}_2^2}{2} \right) \dot{m}$$

A simplified form of the Energy Equation

$$\dot{Q} - \dot{W}_s + \left(\frac{p_1}{\rho} + gz_1 + u_1 + \alpha_1 \frac{\bar{V}_1^2}{2}\right)\dot{m} = \left(\frac{p_2}{\rho} + gz_2 + u_2 + \alpha_2 \frac{\bar{V}_2^2}{2}\right)\dot{m}$$

Dividing by $\dot{m}g$

$$\frac{\dot{W}_p}{\dot{m}g} + \frac{p_1}{\gamma} + z_1 + \alpha_1 \frac{\bar{V}_1^{\,2}}{2g} = \frac{\dot{W}_t}{\dot{m}g} + \frac{p_2}{\gamma} + z_2 + \alpha_2 \frac{\bar{V}_2^{\,2}}{2g} + \frac{u_2 - u_1}{g} - \frac{\dot{Q}}{\dot{m}g}$$

We already know that:

$$h_p = \frac{\dot{W}_p}{\dot{m}g} \qquad \& \qquad h_t = \frac{\dot{W}_t}{\dot{m}g}$$

So we have:

$$\frac{p_1}{\gamma} + \alpha_1 \frac{\bar{V}_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{\bar{V}_2^2}{2g} + z_2 + h_t + \left(\frac{u_2 - u_1}{g} - \frac{\dot{Q}}{\dot{m}g}\right)$$

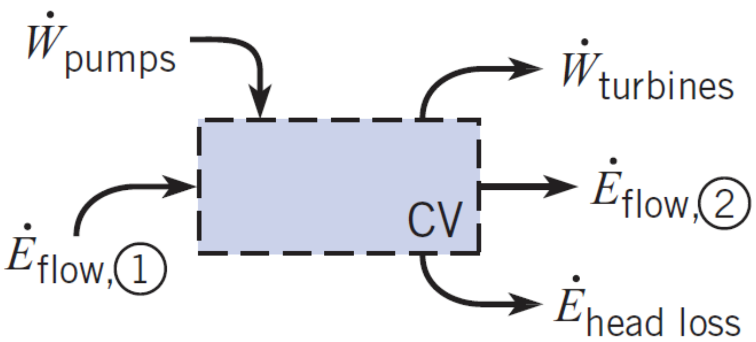
So:

$$\left(\frac{p_1}{\gamma} + \alpha_1 \frac{\bar{V}_1^2}{2g} + z_1\right) + h_p = \left(\frac{p_2}{\gamma} + \alpha_2 \frac{\bar{V}_2^2}{2g} + z_2\right) + h_t + h_L$$

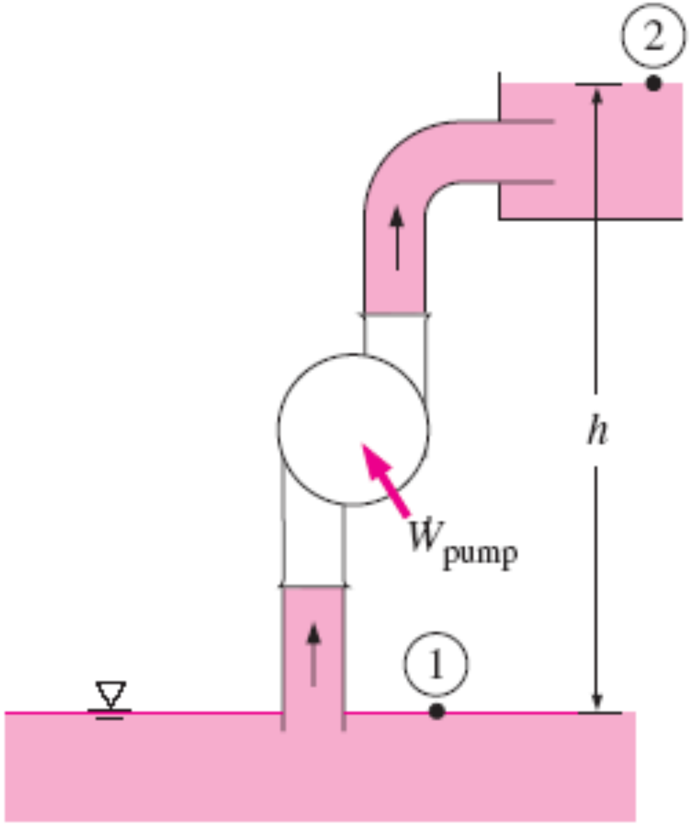
A simplified form of the Energy Equation

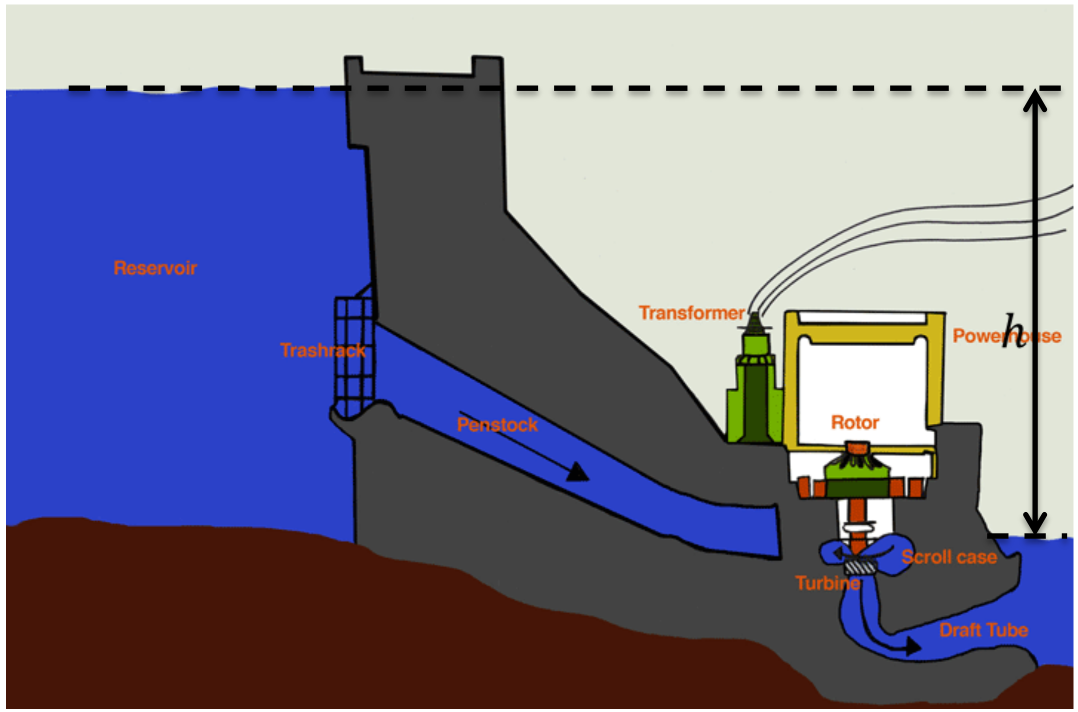
$$\left(\frac{p_1}{\gamma} + \alpha_1 \frac{\bar{V}_1^2}{2g} + z_1\right) + h_p = \left(\frac{p_2}{\gamma} + \alpha_2 \frac{\bar{V}_2^2}{2g} + z_2\right) + h_t + h_L$$

$$\begin{pmatrix} \text{pressure head} \\ \text{velocity head} \\ \text{elevation head} \end{pmatrix} + \begin{pmatrix} \text{pump} \\ \text{head} \end{pmatrix} = \begin{pmatrix} \text{pressure head} \\ \text{velocity head} \\ \text{elevation head} \end{pmatrix} + \begin{pmatrix} \text{turbine} \\ \text{head} \end{pmatrix} + \begin{pmatrix} \text{head} \\ \text{loss} \end{pmatrix}$$



Pump and Turbine Head

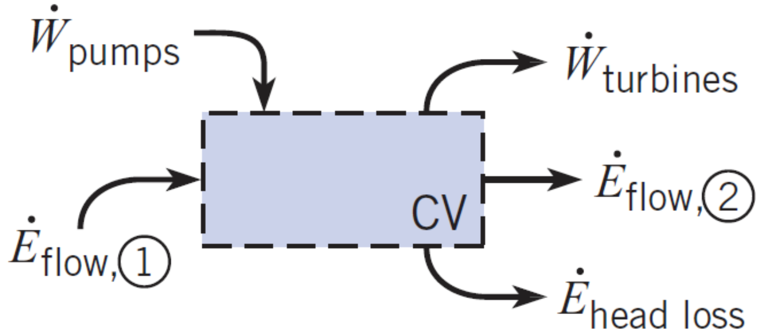




Head

Head has a primary dimension of ***length***, it is related to ***energy*** or ***work.***

$$\text{head} \approx \frac{\text{energy/time or work/time}}{\text{weight/time of flowing fluid}}$$



Pump Head:

$$Pump\ head = h_p = \frac{\dot{W}_P}{\dot{m}g} = \frac{\text{work/time done by pump on flow}}{\text{weight/time of flowing fluid}}$$

Turbine Head:

$$Turbine\ head = h_t = \frac{\dot{W}_t}{\dot{m}g} = \frac{\text{work/time done by flow on turbine}}{\text{weight/time of flowing fluid}}$$

Head Loss:

The conversion of useful mechanical energy to waste thermal energy through viscous action between fluid particles.

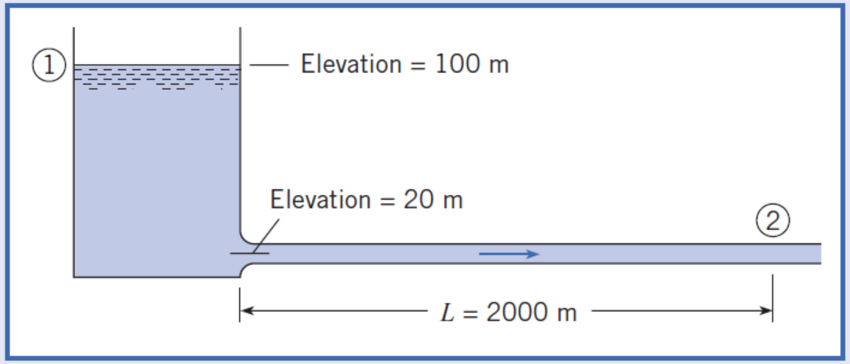
EXAMPLE 7.2

Pressure in a Pipe

A horizontal pipe carries cooling water at 10°C for a thermal power plant from a reservoir as shown. The head loss in the pipe is

$$h_L = \frac{0.02(L/D)V^2}{2g}$$

where L is the length of the pipe from the reservoir to the point in question, V is the mean velocity in the pipe, and D is the diameter of the pipe. If the pipe diameter is 20 cm and the rate of flow is 0.06 m^3/s, what is the pressure in the pipe at $L = 2000$ m. Assume $\alpha_2 = 1$.



Plan

1. Write the energy equation from Eq. (7.29) between section 1 and section 2.
2. Analyze each term in the energy equation.
3. Solve for p_2.

Assumptions:

$$\alpha_2 = 1;$$

Properties:

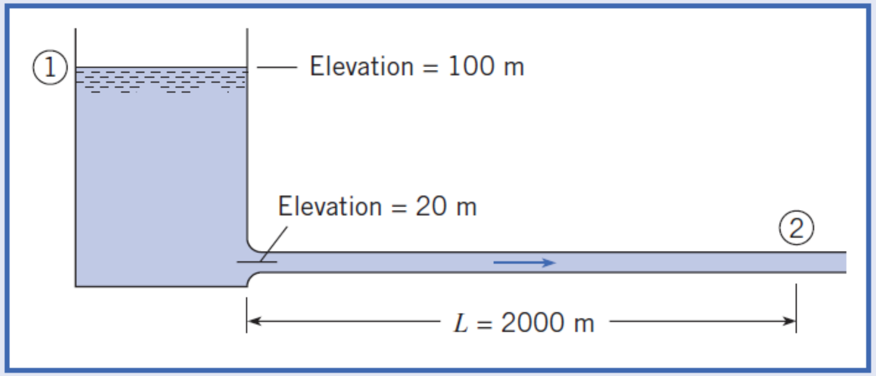
$$\gamma(water\ at\ 10°C) = 9810\ N/m^3$$

We are looking for?

$$p_2$$

EXAMPLE 7.2

Pressure in a Pipe



Plan

1. Write the energy equation from Eq. (7.29) between section 1 and section 2.
2. Analyze each term in the energy equation.
3. Solve for p_2.

1. Energy Equation:

$$\frac{p_1}{\gamma} + \alpha_1 \frac{\bar{V}_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{\bar{V}_2^2}{2g} + z_2 + h_t + h_L$$

2. Term by term analysis:

$p_1 = 0$ (Why?)

$V_1 \cong 0$ (Why?)

$z_1 = 100\ m; z_2 = 20\ m$

$h_p = h_t = 0$ (Why?)

$$V_2 = \frac{Q}{A} = \frac{0.06\frac{m^3}{s}}{\frac{\pi}{4}(0.2\text{ m})^2} = 1.910\frac{m}{s}$$

$$h_L = \frac{0.02\left(\frac{L}{D}\right)V^2}{2g} =$$

$$\frac{0.02*\left(\frac{2000\text{m}}{0.2\text{m}}\right)\left(1.910\frac{\text{m}}{\text{s}}\right)^2}{2\left(9.81\frac{\text{m}}{\text{s}^2}\right)} = 37.2\text{ m}$$

EXAMPLE 7.2

Pressure in a Pipe

1. Energy Equation:

$$\frac{p_1}{\gamma} + \alpha_1 \frac{\bar{V}_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{\bar{V}_2^2}{2g} + z_2 + h_t + h_L$$

2. Term by term analysis:

$$p_1 = 0$$
$$V_1 = 0$$
$$z_1 = 100\ m; z_2 = 20\ m$$
$$h_p = h_t = 0$$
$$V_2 = 1.910 \frac{m}{s}$$
$$h_L = 37.2\ \text{m}$$

3. Combining step 1 and 2:

$$(z_1 - z_2) = \frac{p_2}{\gamma} + \alpha_2 \frac{\bar{V}_2^2}{2g} + h_L$$

$$80m = \frac{p_2}{\gamma} + 1.0 \frac{\left(1.910 \frac{m}{s}\right)^2}{2\left(9.81 \frac{m}{s^2}\right)} + 37.2$$

$$p_2 = \gamma(42.6\ m) = \left(9810 \frac{N}{m^3}\right)(42.6\ m)$$

$$\boldsymbol{p_2 = 418\ kPa}\ \text{(gage)}$$

Efficiency

Efficiency is defined as:

$$\eta \equiv \frac{power\ output\ from\ a\ machine\ or\ system}{power\ input\ to\ a\ machine\ or\ system} = \frac{P_{output}}{P_{input}}$$

Pumps transfer electrical power (P_{in}) to the fluid as mechanical (shaft) power($\dot{W}_p$) :

$$\eta_p = \frac{P_{out}}{P_{in}} = \frac{\dot{W}_p}{P_{in}} = \frac{\gamma Q h_p}{P_{in}} = \frac{\dot{m} g h_p}{P_{in}}$$

Turbines converts power of the flow ($\dot{W}_t$) to electrical power (P_{out}):

$$\eta_t = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{\dot{W}_t} = \frac{P_{out}}{\gamma Q h_t} = \frac{P_{out}}{\dot{m} g h_t}$$

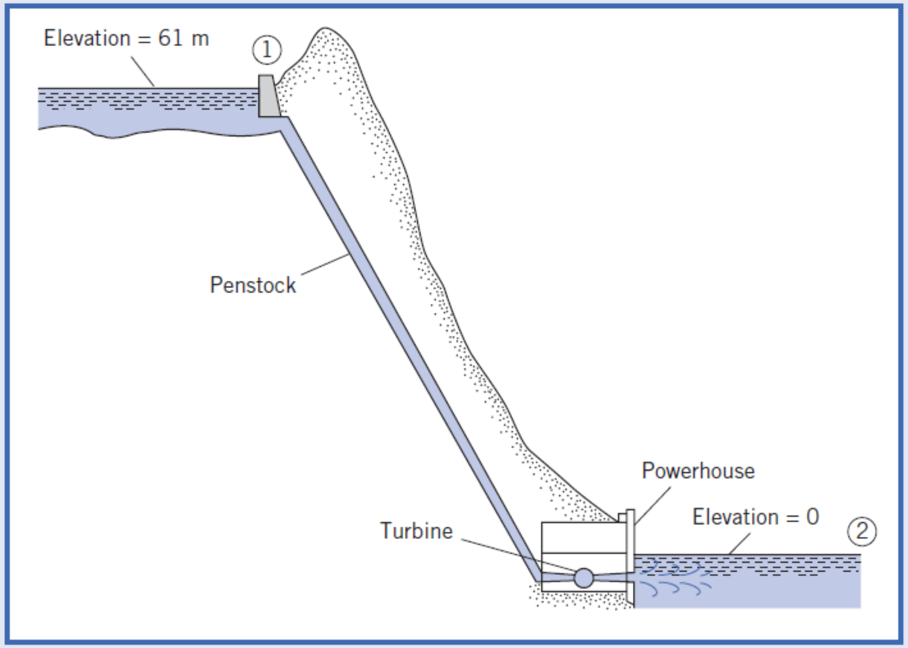
In general, P, $\dot{W}_p$ and $\dot{W}_t$ can be expressed as:

$$P = \dot{m} g h$$

EXAMPLE 7.4

Power produced by a Turbine:

At the maximum rate of power generation, a small hydroelectric power plant takes a discharge of 14.1 m^3/s through an elevation drop of 61 m. The head loss through the intakes, penstock, and outlet works is 1.5 m. The combined efficiency of the turbine and electrical generator is 87%. What is the rate of power generation?



Properties:

$$\gamma(water\ at\ 10°C) = 9810\ N/m^3$$

We are looking for?

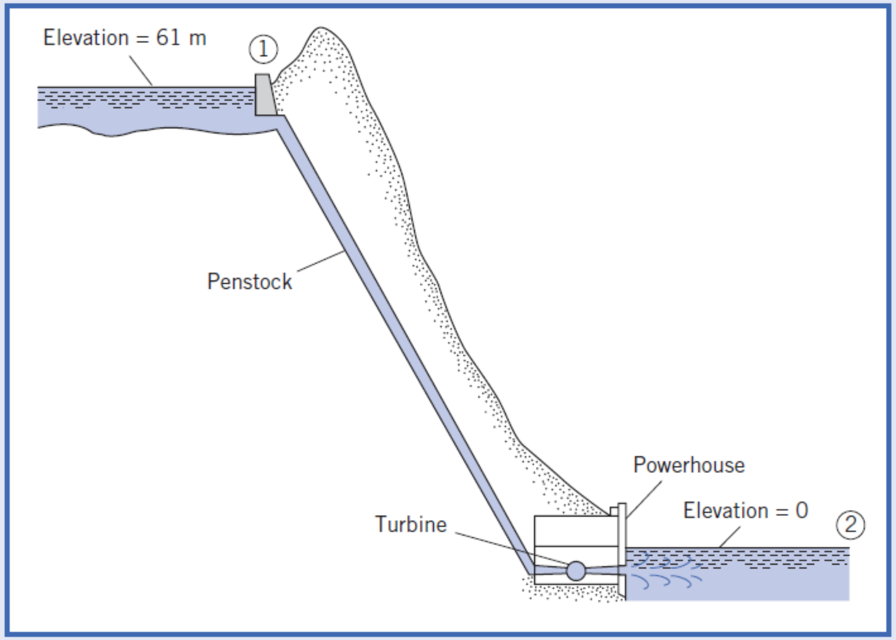
$$P_{input\ to\ turbine}$$

&

$$P_{output\ from\ Generator}$$

EXAMPLE 7.4

Power produced by a Turbine:



Plan

1. Write the energy equation (7.29) between section 1 and section 2.
2. Analyze each term in the energy equation.
3. Solve for the head of the turbine h_t.
4. Find the input power to the turbine using the power equation (7.30a).
5. Find the output power from generator by using the efficiency equation (7.32).

1. Energy Equation:

$$\frac{p_1}{\gamma} + \alpha_1 \frac{\bar{V}_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{\bar{V}_2^2}{2g} + z_2 + h_t + h_L$$

2. Term by term analysis:

$V_1 \cong 0$ (Why?)

$V_2 \cong 0$ (Why?)

$p_1 = p_2 = 0$ (Why?)

$h_p = 0$

$z_1 = 61\ m; \quad z_2 = 0;$

$h_L = 1.5\ \mathrm{m}$

EXAMPLE 7.4

Power produced by a Turbine:

1. Energy Equation:

$$\frac{p_1}{\gamma} + \alpha_1 \frac{\bar{V}_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{\bar{V}_2^2}{2g} + z_2 + h_t + h_L$$

2. Term by term analysis:

$$V_1 \cong 0$$
$$V_2 \cong 0$$
$$p_1 = p_2 = 0$$
$$h_p = 0$$
$$z_1 = 61\,m; \quad z_2 = 0;$$
$$h_L = 1.5\text{ m}$$

3. **Combining step 1 and 2:**

$$h_t = (z_1 - z_2) - h_L = (61\,m) - (1.5\,m) = 59.5\,m$$

4. **Power equation:**

$$P_{\text{input to turbine}} = \dot{W}_t = \dot{m}gh_t = \gamma Q h_t$$

$$= \left(9810\,\frac{N}{m^3}\right)\left(14.1\,\frac{m^3}{s}\right)(59.5\,m)$$

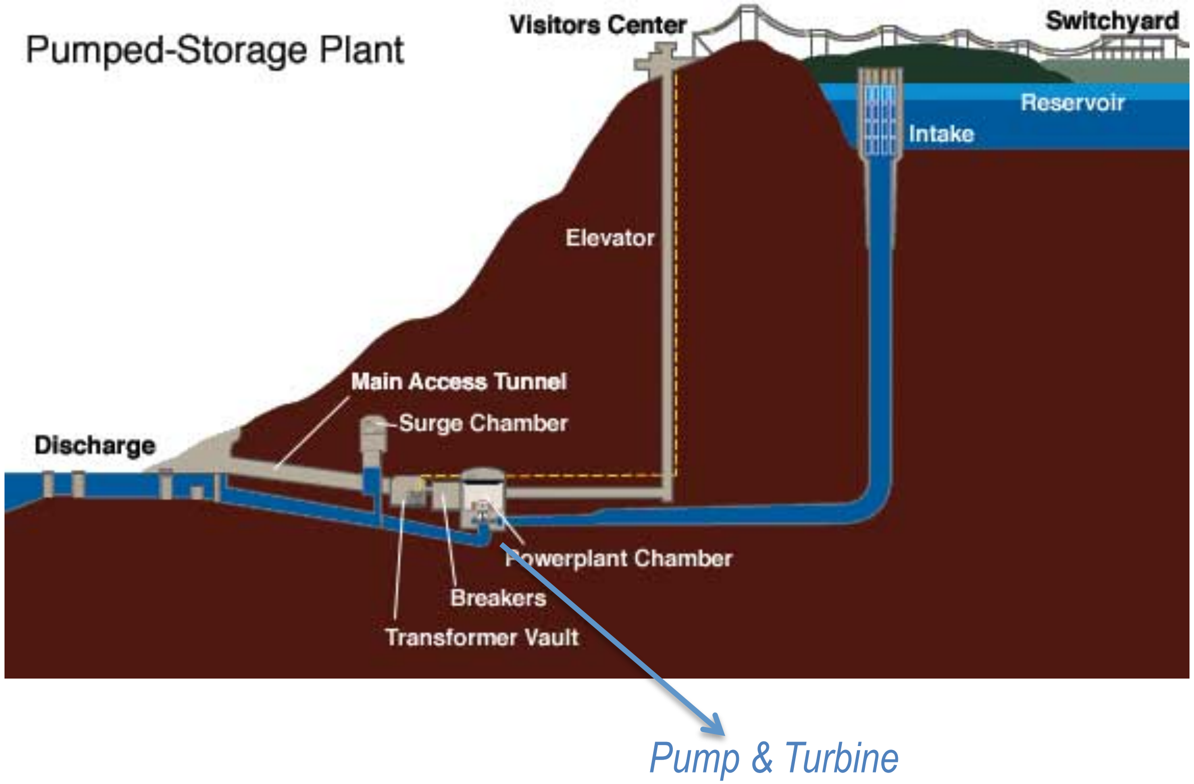
$$= 8.23\,MW$$

5. **Efficiency equation**

$$P_{output\ from\ generator} = \eta P_{input\ to\ turbine}$$

$$= 0.87(8.23MW) = \mathbf{7.16\,MW}$$

Example: Pumped Storage



- Sometimes the water is pumped up (where it is stored as potential energy): ***When?***
- Other times that potential energy is used and converted into electricity : ***When?***
- Considering the head losses and non-perfect efficiency of the pump/turbine, why is it useful?

The Bernoulli and Energy Equations:

The Bernoulli Equation

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

Assumptions:

- Steady flow
- Incompressible flow
- *Inviscid* flow
- *Without* additional Energy added or extracted (without Pump or Turbine)

The Energy Equation

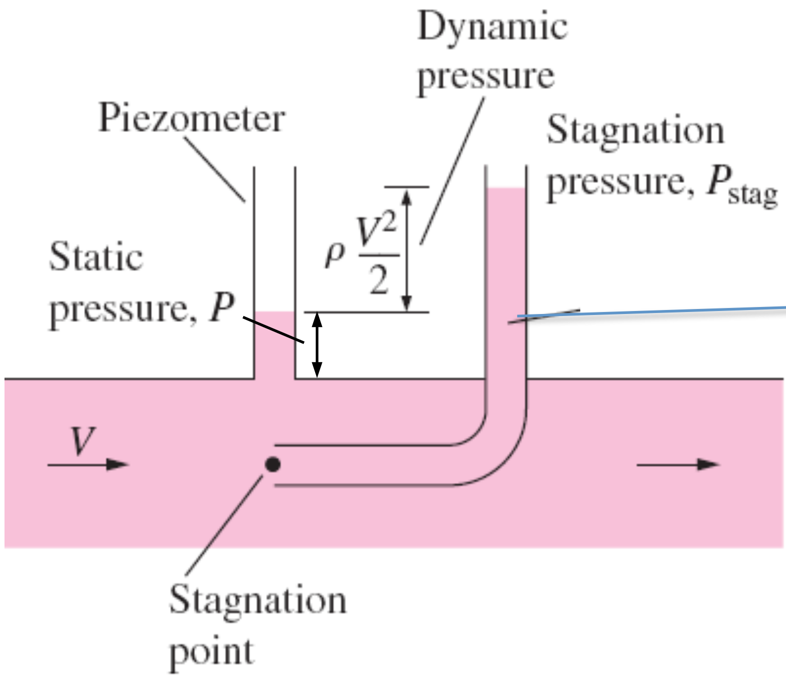
$$\left(\frac{p_1}{\gamma} + \alpha_1 \frac{\bar{V}_1^2}{2g} + z_1\right) + h_p = \left(\frac{p_2}{\gamma} + \alpha_2 \frac{\bar{V}_2^2}{2g} + z_2\right) + h_t + h_L$$

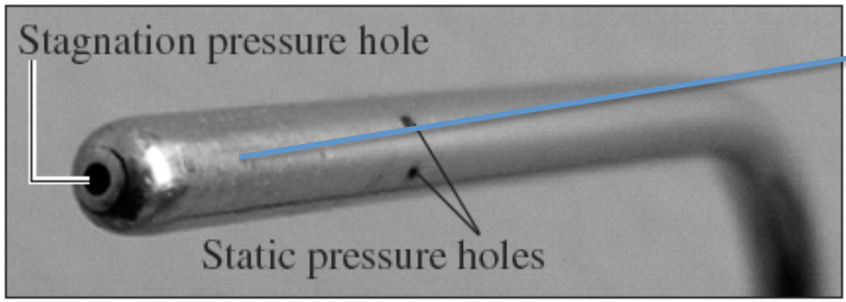
Assumptions:

- Steady flow
- Incompressible flow
- *Viscous* flow (→ head losses)
- *With* additional Energy added or extracted (by Pump or Turbine)

Static and Dynamic Pressure

Piezometer and Stagnation tube





- **Piezometer** measures static pressure (*p*)

- **Stagnation tube** measures 'stagnation pressure' = static pressure + dynamic pressure

$$p_{stag} = p + \rho \frac{V^2}{2}$$

- **Pitot tube** measures the dynamic pressure (difference between stagnation pressure and the static pressure) and, therefore, the velocity

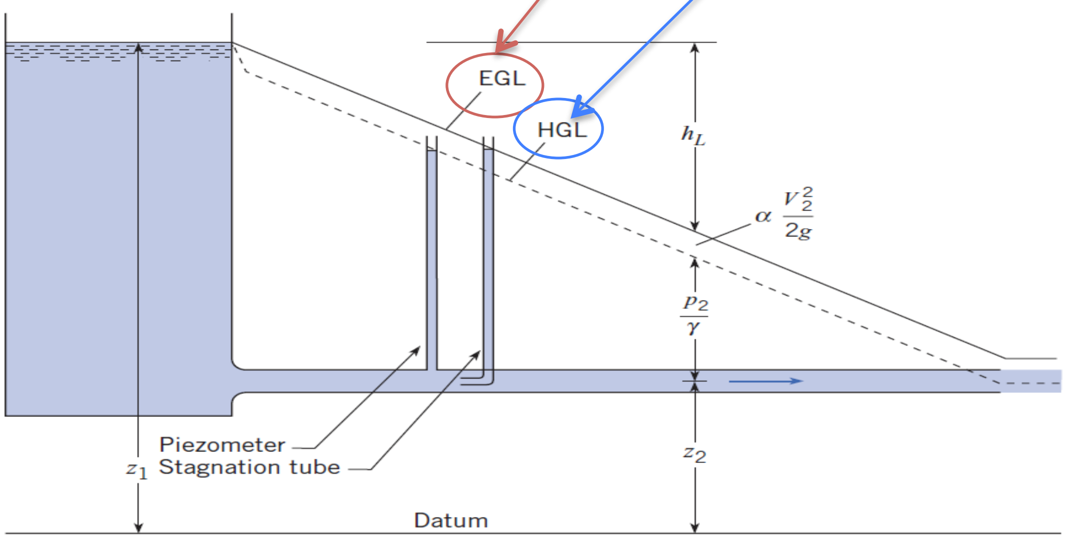
Hydraulic and Energy Grade Lines

- Energy Grade Lines (EGL) is a line that indicates the total head at each location in a system.

$$\text{EGL} = \begin{pmatrix}\text{velocity}\\ \text{head}\end{pmatrix} + \begin{pmatrix}\text{pressure}\\ \text{head}\end{pmatrix} + \begin{pmatrix}\text{elevation}\\ \text{head}\end{pmatrix} = \alpha\frac{V^2}{2g} + \frac{p}{\gamma} + z = \begin{pmatrix}\text{total}\\ \text{head}\end{pmatrix}$$

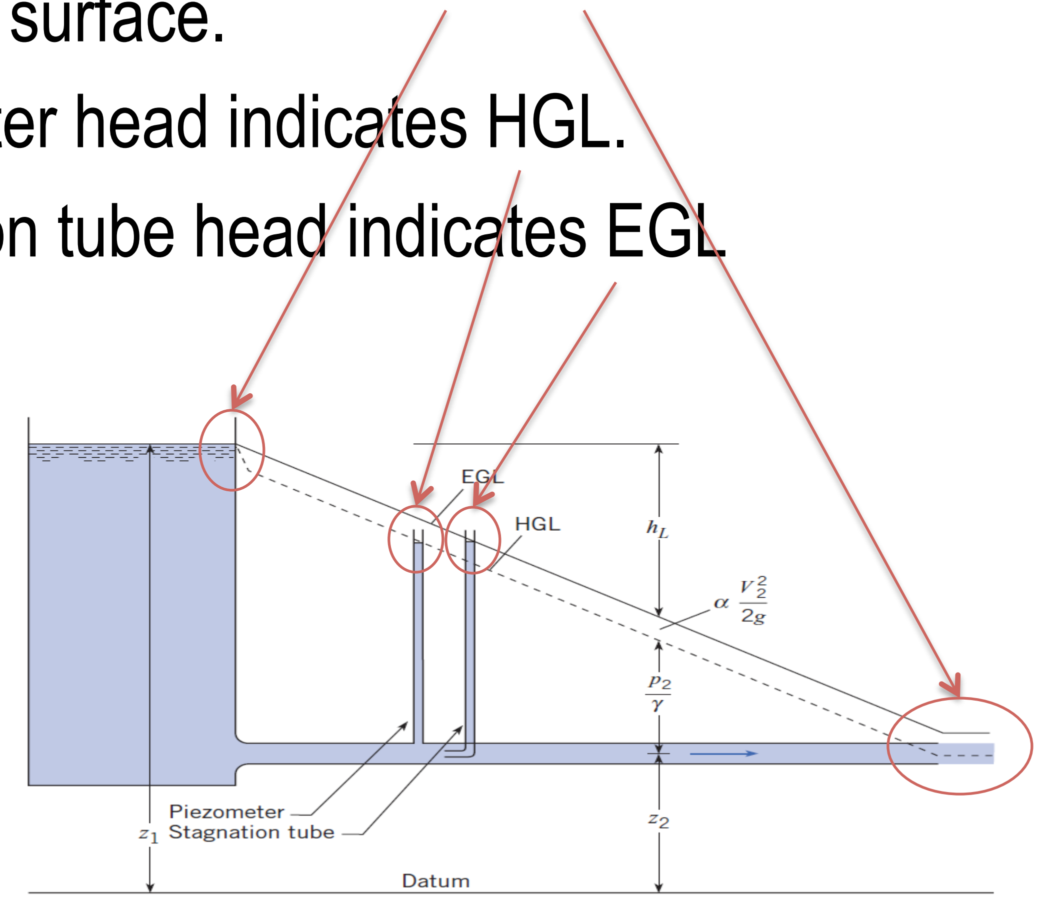
- Hydraulic Grade Line (HGL) is a line that indicates the piezometric head at each location in a system.

$$\text{HGL} = \begin{pmatrix}\text{pressure}\\ \text{head}\end{pmatrix} + \begin{pmatrix}\text{elevation}\\ \text{head}\end{pmatrix} = \frac{p}{\gamma} + z = \begin{pmatrix}\text{piezometric}\\ \text{head}\end{pmatrix}$$



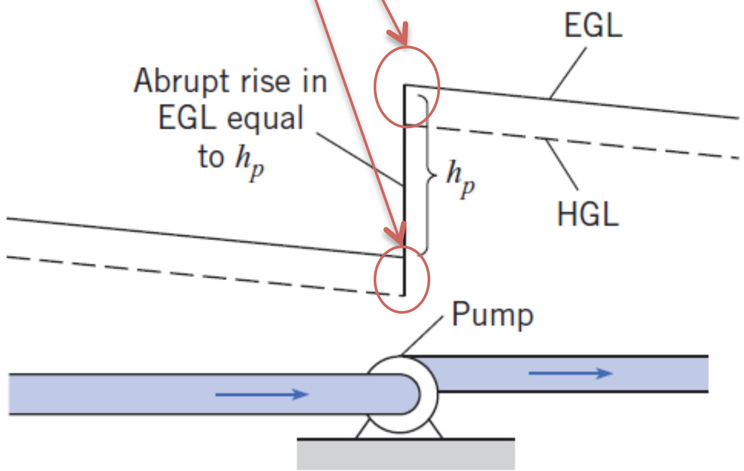
Guidelines for HGL and EGL

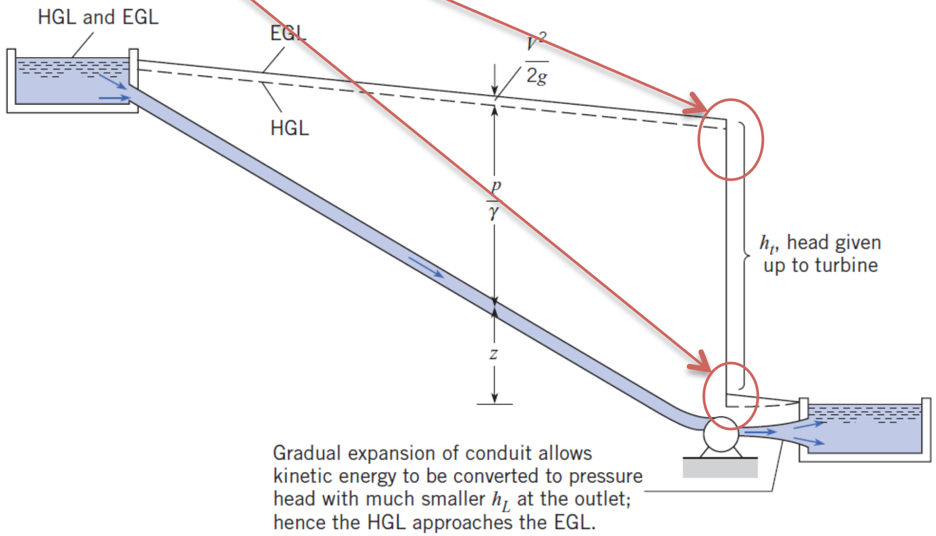
- In a lake or reservoir, the HGL and EGL will coincide with the liquid surface.
- Piezometer head indicates HGL.
- Stagnation tube head indicates EGL.



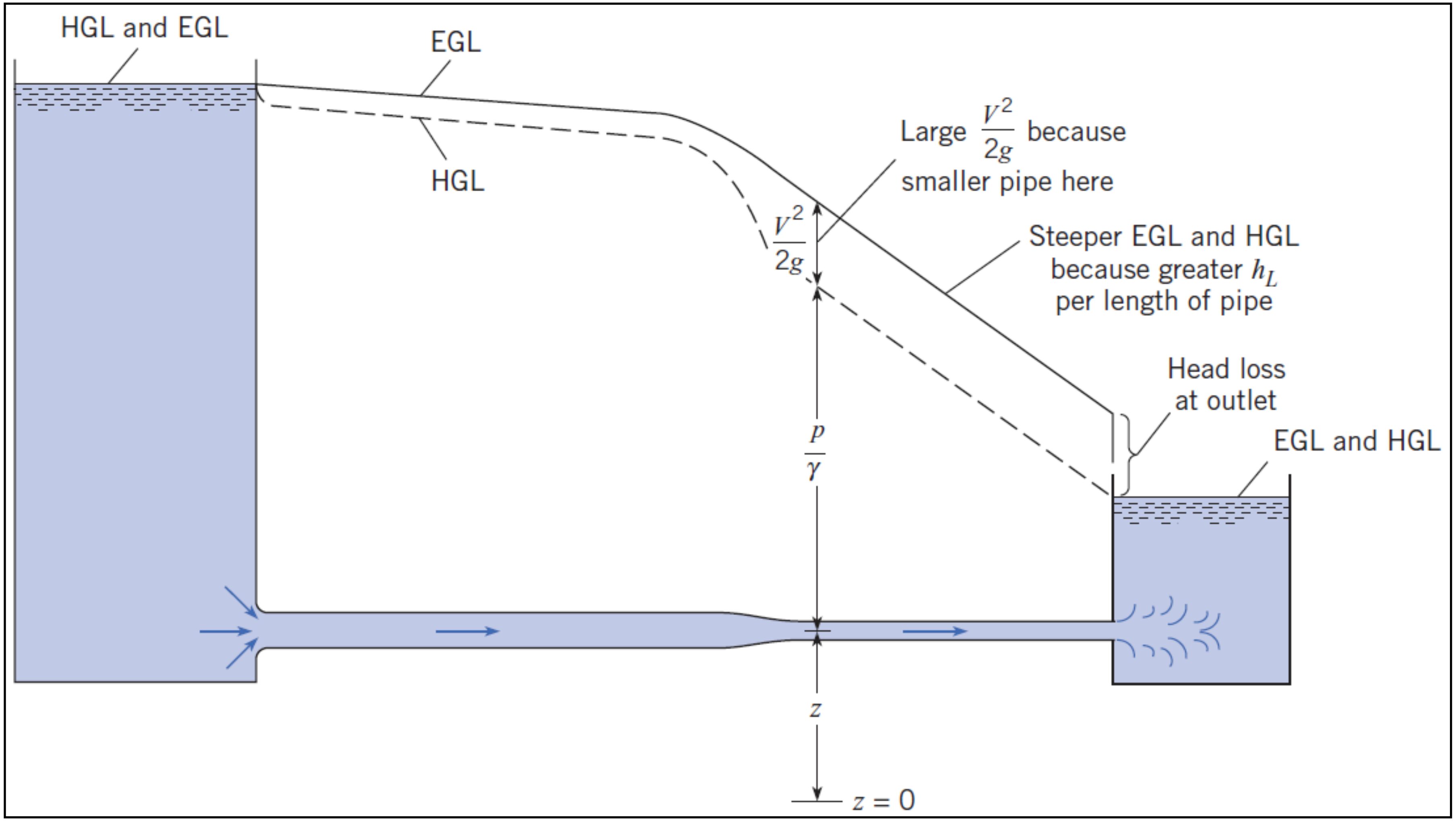
Guidelines for HGL and EGL

- Pump rises HGL and EGL.
- Turbine drops HGL and EGL.





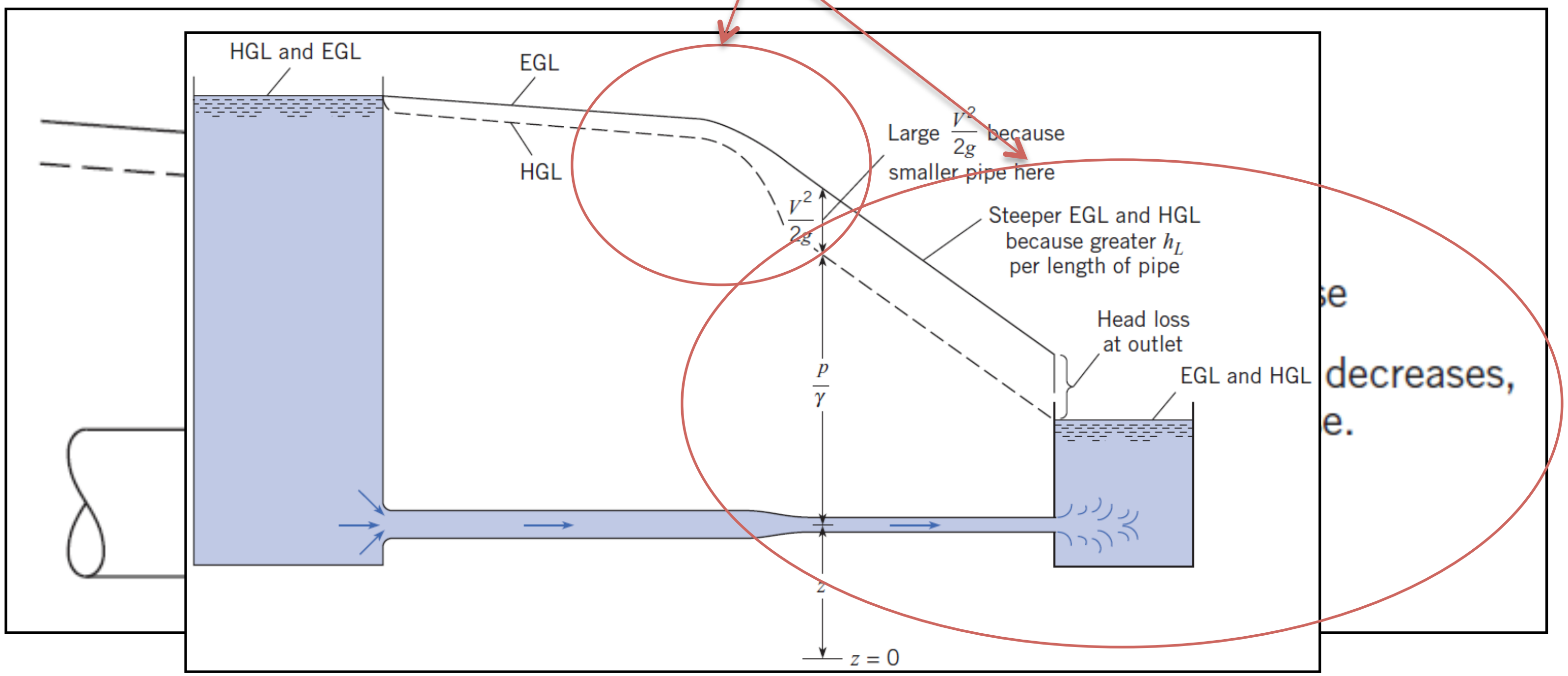
Guidelines for HGL and EGL



Guidelines for HGL and EGL

When a flow passes a Nozzle or when passage changes diameter:

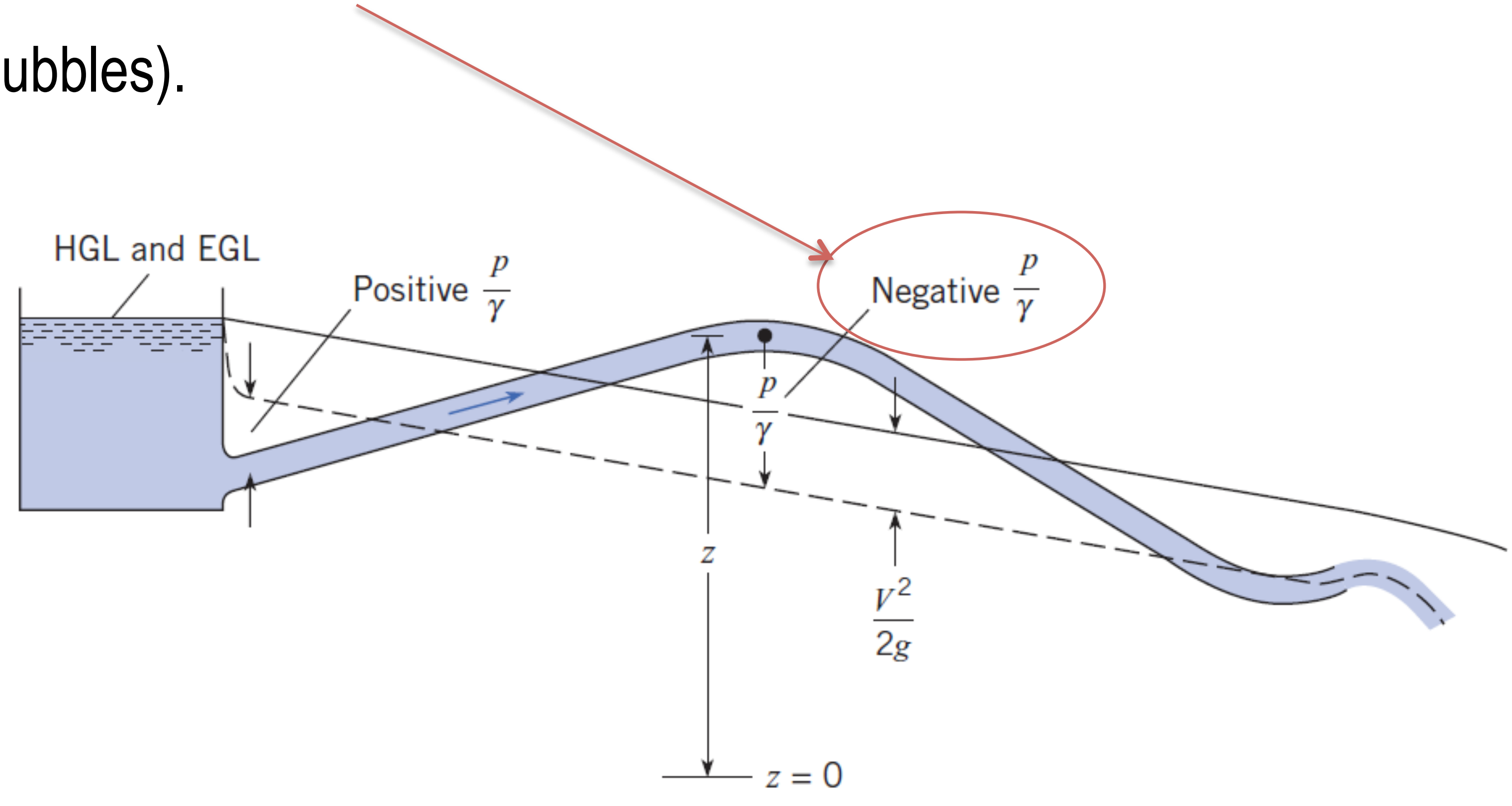
- HGL changes due to velocity changes.
- EGL changes also because head loss per length is larger for a flow in a conduit with larger velocity.



Guidelines for HGL and EGL

Cavitation:

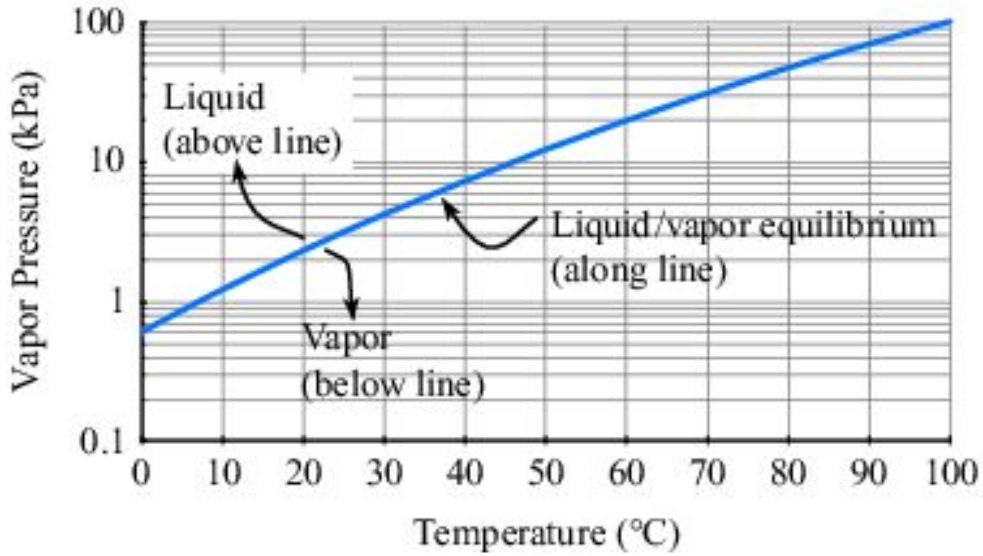
- If the HGL falls below the pipe, then $\frac{p}{\gamma}$ is negative, indicating subatmospheric pressure and a potential location of cavitation (formation of bubbles).

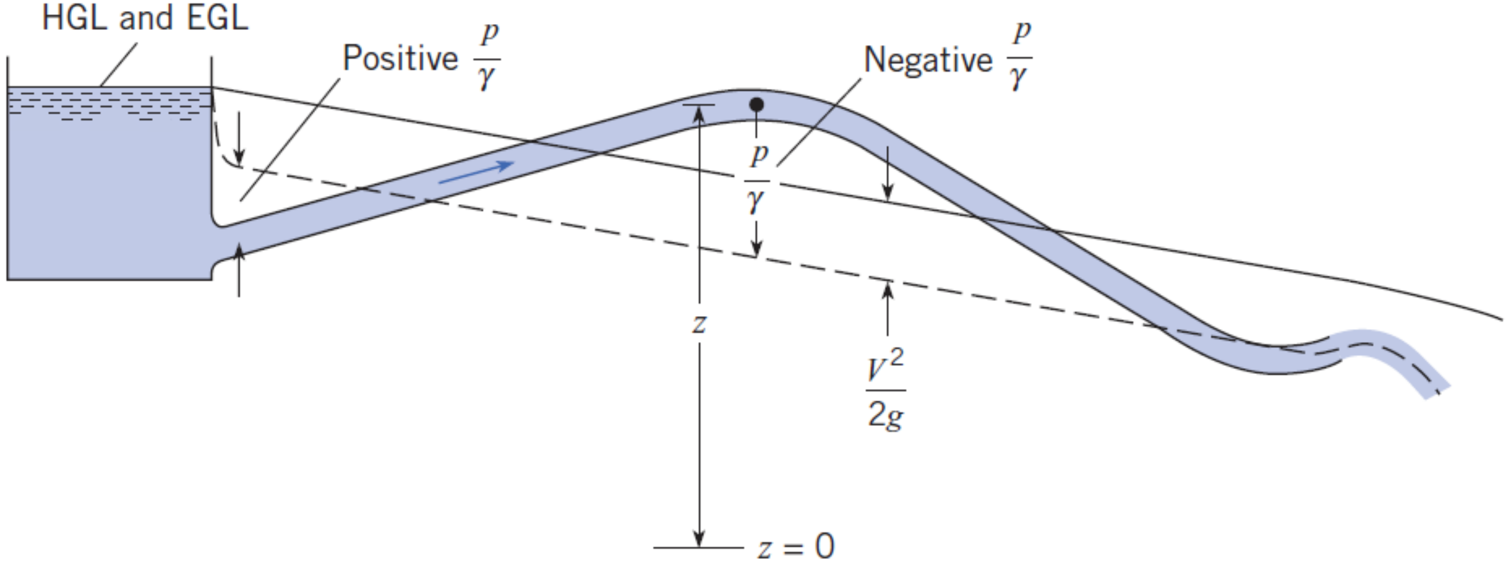


Guidelines for HGL and EGL

Cavitation:

If pressure becomes lower than the vapor pressure (see Fig. 2.21: vapor pressure for water), then cavitation occurs





EXAMPLE 7.6

EGL and HGL

A pump draws water (10^o C) from a reservoir., where the water-surface elevation is 160 m, and forces the water through a pipe 1525 m long and 0.3 m in diameter. This pipe then discharges the water into a reservoir with water-surface elevation of 190 m. the flow rate is 0.2 m^3/s, and the head loss in the pipe is given by

$$h_L = 0.01\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right)$$

Determine the head supplied by the pump, h_p, and the power supplied to the flow, and draw the HGL and EGL for the system. Assume that the pipe is horizontal and is 155 m in elevation.

We are looking for?

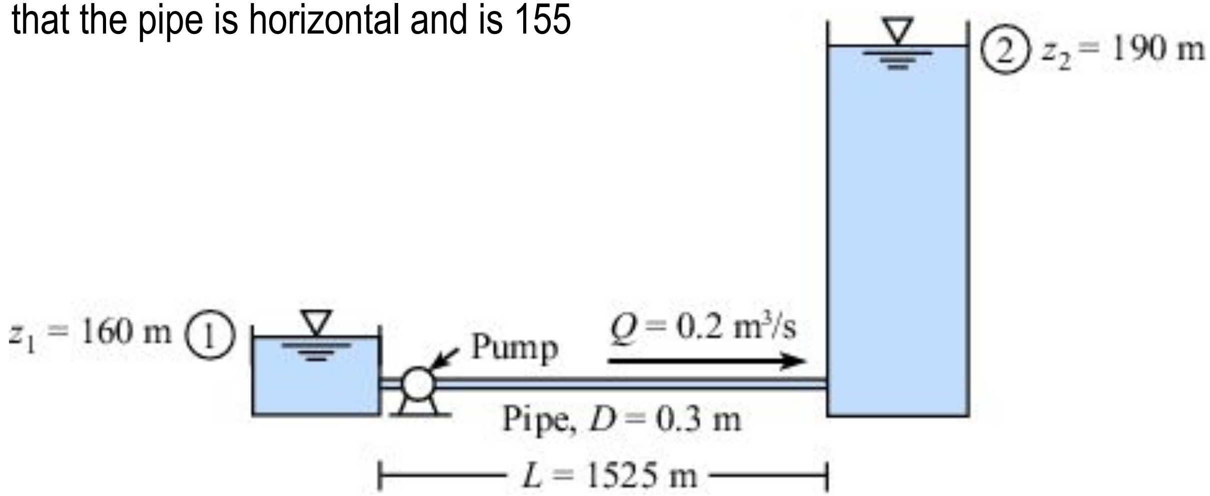
Pump head (in m)

Power (in kW) supplied to the flow

Draw HGL and EGL

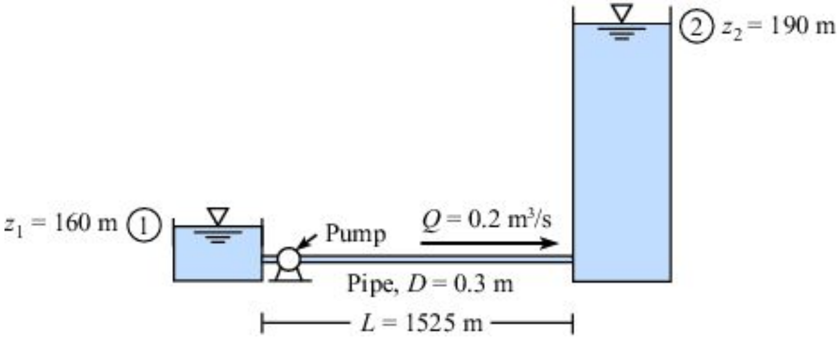
Properties:

$$Water\left(10^o C\right) \Rightarrow \gamma = 9810 \text{ N/m}^3$$



EXAMPLE 7.6

EGL and HGL for a system



Plan

1. Apply the energy equation (7.29) between sections 1 and section 2.
2. Calculate terms in the energy equation.
3. Find the power by applying the power equation (7.30a).
4. Draw the HGL and EGL by using the tips given on p. 270.

1. Apply the energy equation between sections 1 and 2:

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p =$$

$$\frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

2. Calculate terms in the energy equation:

$$V_1 \approx V_2 \approx 0 \quad (Why?)$$

$$p_1 \approx p_2 \approx 0 \quad (Why?)$$

$$h_t = 0$$

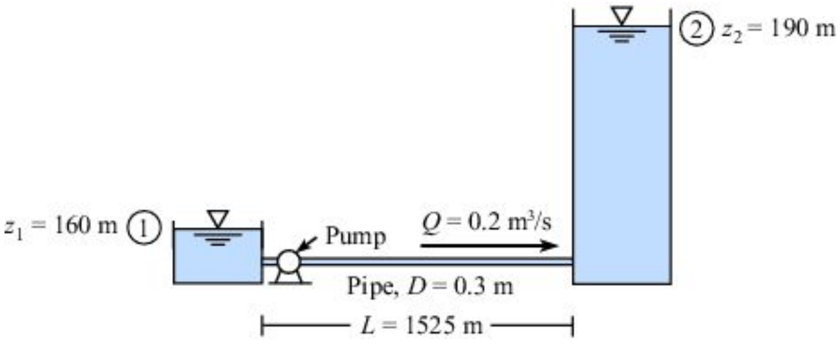
$$h_p = z_2 - z_1 + h_L$$

Calculate V using the flow rate equation:

$$V = \frac{Q}{A} = \frac{0.2\,m^3/s}{(\pi/4)(0.3\,m)^2} = 2.83\,m/s$$

EXAMPLE 7.6

EGL and HGL for a system



Plan

1. Apply the energy equation (7.29) between sections 1 and section 2.
2. Calculate terms in the energy equation.
3. Find the power by applying the power equation (7.30a).
4. Draw the HGL and EGL by using the tips given on p. 270.

The pump provides:

1.The energy to lift the fluid to a higher elevation

2.The energy to overcome head loss

$$h_L = 0.01\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right) = 0.01\left(\frac{1525\,m}{0.3\,m}\right)\left(\frac{(2.83\,m/s)^2}{2\times(9.81\,m/s^2)}\right)$$

$$= 2.75\,m$$

$$h_p = (z_2 - z_1) + h_L = (190\,m - 160\,m) + 20.75\,m$$

$$= 50.75\,m$$

EXAMPLE 7.6

EGL and HGL for a system

1. Apply the energy equation between sections 1 and 2

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p =$$

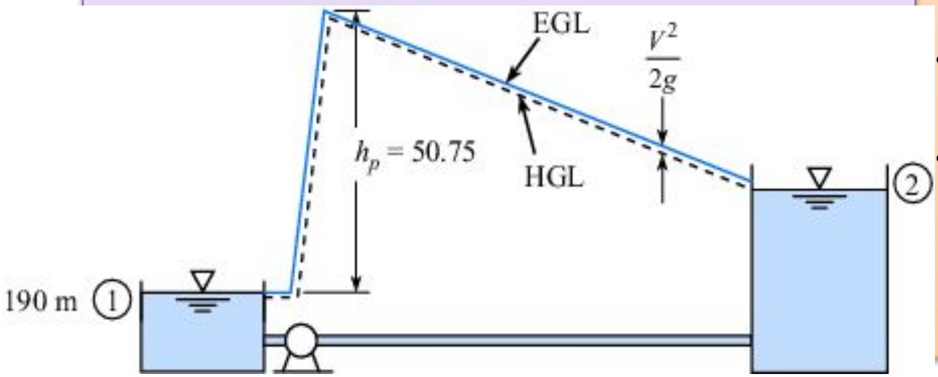
$$\frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

2. Calculate terms in the energy equation

$$V_1 \approx V_2 \approx 0 \qquad h_L = 2.75\ m$$

$$p_1 \approx p_2 \approx 0 \qquad h_P = 50.75\ m$$

$$h_t = 0$$



3. Power

$$\dot{W}_p = \gamma Q h_p = \left(9810 \frac{N}{m^3}\right)\left(0.2 \frac{m^3}{s}\right)(50.75 m)\left(\frac{W.s}{N.m}\right)$$

$$= 99.5 kW$$

4. HGL and EGL

- From Tip 1 on p. 233, locate the HGL and EGL along the reservoir surfaces.
- From Tip 2, sketch in a head rise of 50.75 m corresponding to the pump.
- From Tip 3, sketch the EGL from the pump outlet to the reservoir surface. Use the fact that the head loss is 20.75 m. Also, sketch EGL from the reservoir on the left to the pump inlet. Show a small head loss.
- From Tip 4, sketch the HGL below the EGL by a distance of $V^2 / 2g \approx 0.5 m$
- From Tip 5, check the sketches to ensure that EGL and HGL are decreasing in the direction of flow (except at the pump).